Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Write down the value of m, given that $3^4 \times 3^5 = 3^m$

$3^{4+5} = 3^9$

$m =$ 9

(1)

(b) Write down the value of n, given that $(5^3)^7 = 5^n$

$5^{3\times 7} = 5^{21}$

$n =$ 21

(1)

(c) Find the value of p, given that $\dfrac{7^8 \times 7^2}{7^p} = 7^6$

$(8+2) - p = 6$

$10 - 6 = p$

$p = 4$

$p =$ 4

(2)

(Total for Question 1 is 4 marks)

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2 Here are two rectangles, rectangle *A* and rectangle *B*.

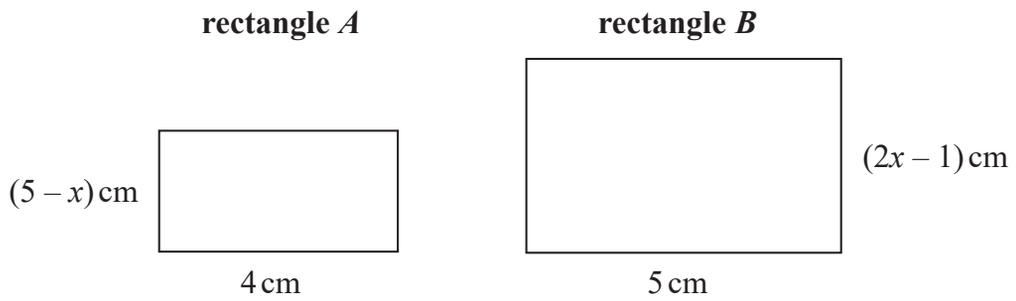

Diagram **NOT** accurately drawn

The area of rectangle *B* is twice the area of rectangle *A*.

Work out the value of *x*.
Show your working clearly.

A

area $4(5-x)$

$= 20 - 4x$

B

$5(2x-1)$

$= 10x - 5$

$B = 2A$

$10x - 5 = 2(20 - 4x)$

$10x - 5 = 40 - 8x$

$18x = 45$

$x = \frac{45}{18} = 2.5$

$x =$ 2.5

(Total for Question 2 is 4 marks)

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3 The table gives information about the amounts of money, in euros, that 70 of Anjali's friends spent last Saturday.

Money spent (S euros)	Frequency
$0 < S \leqslant 8$	6
$8 < S \leqslant 16$	14
$16 < S \leqslant 24$	19
$24 < S \leqslant 32$	25
$32 < S \leqslant 40$	6

4 12 20 28 36 70

One of Anjali's 70 friends is going to be chosen at random.

(a) Find the probability that this friend spent more than 24 euros last Saturday.

25 + 6 = 31

$\frac{31}{70}$

.....

(1)

(b) Work out an estimate for the mean amount of money spent by Anjali's friends last Saturday.
Give your answer correct to 2 decimal places.

4 x 6 = 24

12 x 14 = 168

20 x 19 = 380

28 x 25 = 700

36 x 6 = 216

1488

1488 ÷ 70

= 21.257...

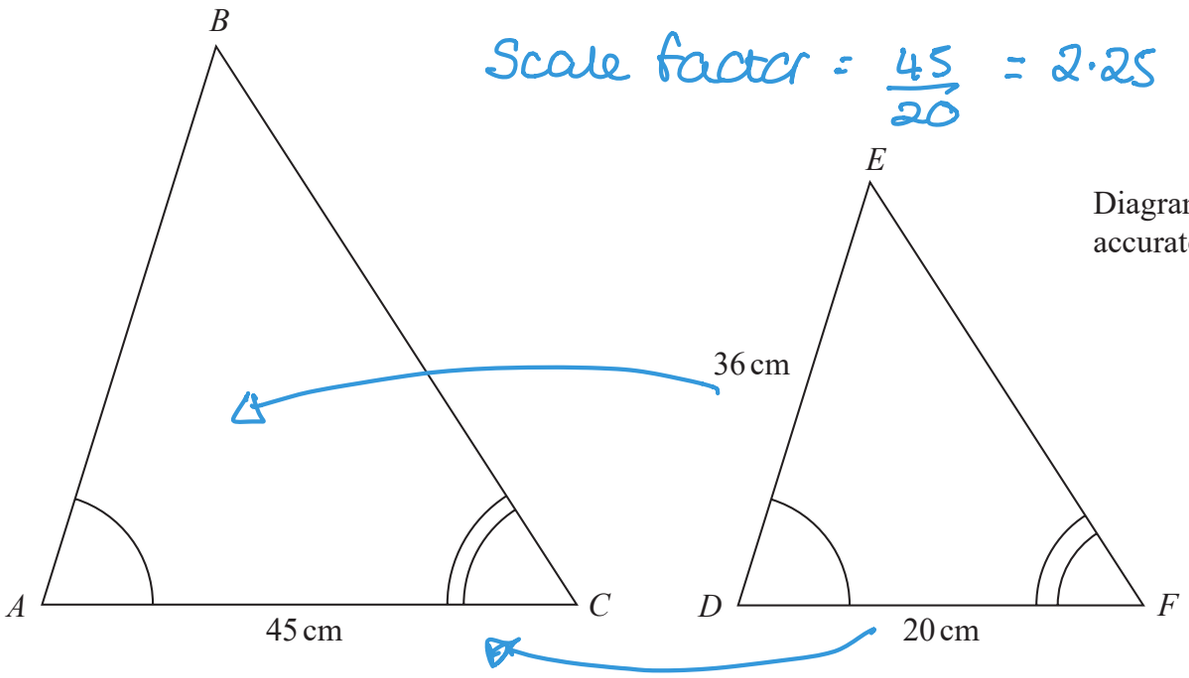
(2dp)

21.26 ..... euros

(4)

(Total for Question 3 is 5 marks)

4 *ABC* and *DEF* are similar triangles.



Scale factor = $\frac{45}{20}$ = 2·25

Diagram **NOT** accurately drawn

(a) Work out the length of *AB*.

36 x 2·25

81 cm

(2)

Given that *BC* = 54 cm,

(b) work out the length of *EF*.

54 ÷ 2·25

24 cm

(2)

(Total for Question 4 is 4 marks)

5 The diagram shows a regular octagon *ABCDHIJK* and a pentagon *DEFGH*.

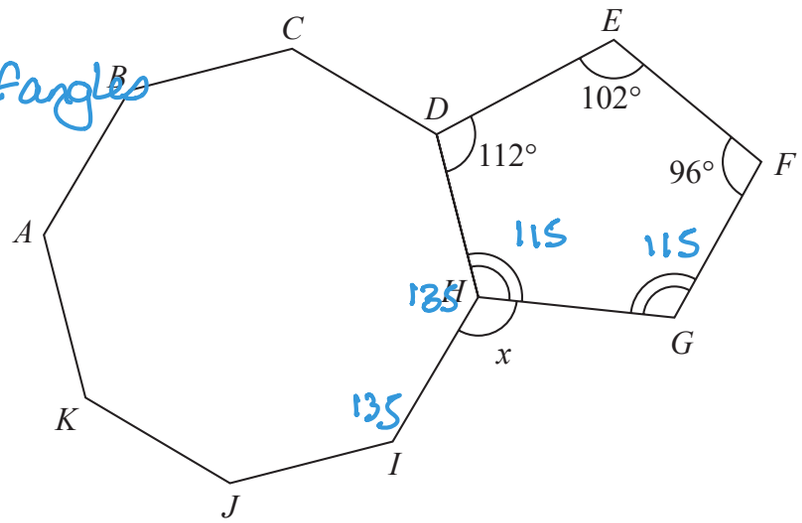


Diagram **NOT** accurately drawn

Angle *GHD* = angle *FGH*.

Work out the size of the angle marked *x*.
Show your working clearly.

sides sum of angles
4 360
5 540
6 720
7 900
8 1080

DEFGH

540 - (112 + 102 + 96) = 230

230 ÷ 2 = 115

Octagon

1080 ÷ 8 = 135

x

360 - (135 + 115)

= 110

110 °

(Total for Question 5 is 5 marks)

6 Victor buys 12 bottles of apple juice for a total cost of $21
Victor sells all 12 bottles at $2.45 each bottle.

Work out Victor's percentage profit.

sells $12 \times 2.45 = 29.40$

Profit $29.40 - 21 = 8.40$

$\% = \frac{8.40}{21} \times 100$

$= 40$

40 %

(Total for Question 6 is 3 marks)

7 Ali and Badia each have 25 000 dollars to invest.

Cyclone Bank	Tornado Bank
Invest 25 000 dollars 4.5% compound interest per year for 3 years	Invest 25 000 dollars Receive 1150 dollars interest each year for 3 years

Ali invests in the Cyclone Bank for 3 years.
Badia invests in the Tornado Bank for 3 years.

By the end of the 3 years, Ali will have received more interest than Badia.

How much more?
Show your working clearly.
Give your answer correct to the nearest dollar.

Cyclone

25000×1.045^3

$= 28529.15....$

interest $= 3529.15$

Tornado

1150×3

$= 3450$

Difference $= 3529.15 - 3450$

$= 79.15$

↑

nearest dollar

79 dollars

(Total for Question 7 is 4 marks)

8 (a) Simplify $(3x^2y)^0$

1

(1)

(b) (i) Factorise $x^2 - 5x - 36$

4, 9 4 − 9

$(x + 4)(x - 9)$

$(x + 4)(x - 9)$

(2)

(ii) Hence solve $x^2 - 5x - 36 = 0$

$(x + 4)(x - 9) = 0$

-4 9

$x = -4$ $x = 9$

(1)

(Total for Question 8 is 4 marks)

9 A rainwater tank contains 2.4×10^7 raindrops.
The rainwater tank also contains 1.75×10^6 bacteria.

(a) Work out the number of bacteria per raindrop in the tank.
Give your answer in standard form correct to 2 significant figures.

$1.75 \times 10^6 \div 2.4 \times 10^7$

$= 0.072916...$

(2sf.)

$= 0.073$

in s.form

7.3×10^{-2}

7.3×10^{-2}

(3)

A drop of rainwater contains 5.01×10^{21} atoms.

In a drop of rainwater the number of atoms is 3 times the number of molecules.

(b) Work out the number of molecules in the rainwater tank.
Give your answer in standard form correct to one significant figure.

rain → atoms = 3 × molecules

tank contains: $2.4 \times 10^7 \times 5.01 \times 10^{21}$ atoms

molecules $= 1.2024 \times 10^{29}$

$\div 3$

$= 4.008 \times 10^{28}$

(1sf)

4×10^{28} molecules

(2)

(Total for Question 9 is 5 marks)

10 ABC is an isosceles triangle with $BA = BC$.

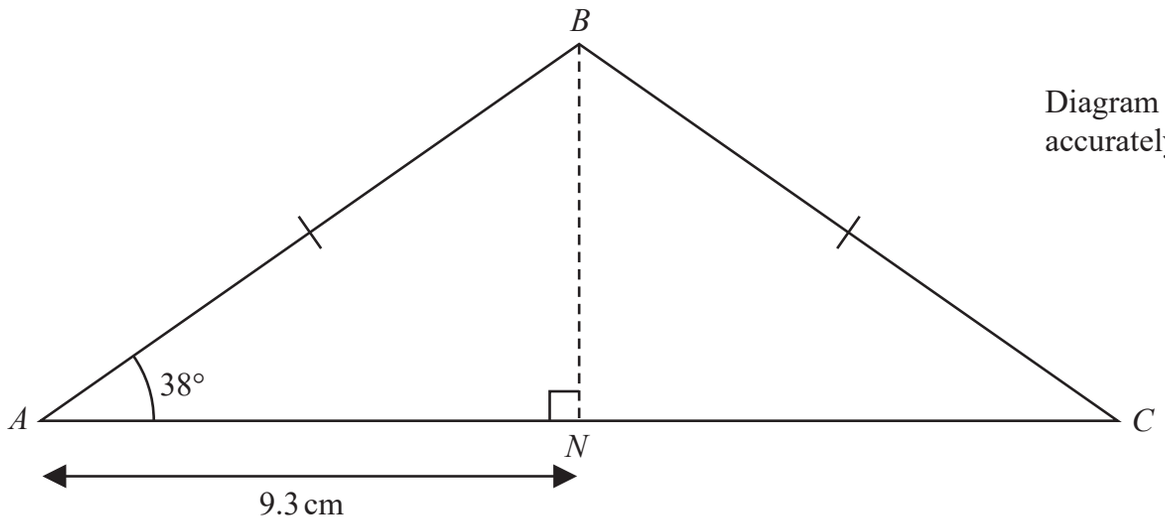


Diagram **NOT** accurately drawn

N is the point on AC such that $AN = 9.3$ cm and BN is perpendicular to AC.

Work out the perimeter of triangle ABC.
Give your answer correct to 3 significant figures.

$\cos 38 = \frac{9.3}{AB}$

$AB = \frac{9.3}{\cos 38} = 11.8018...$

Perimeter $= 2 \times 11.8018... + 2 \times 9.3$

$= 42.2037..$

42.2 ..... cm

(Total for Question 10 is 4 marks)

11

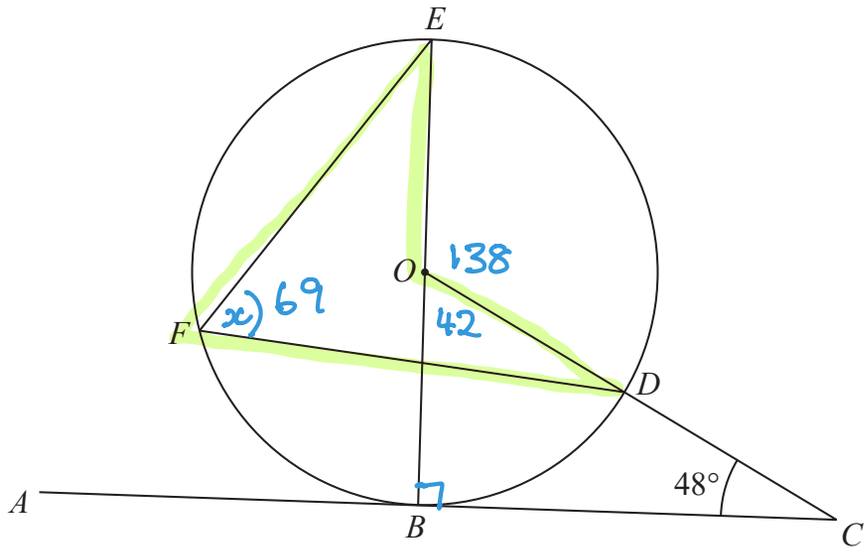


Diagram **NOT** accurately drawn

B, *D*, *E* and *F* are points on a circle, centre *O*.
ABC is a tangent to the circle.
ODC is a straight line.

BOE is a diameter of the circle.

Angle *BCD* = 48°

Find the size of angle *DFE*.

180 - (90 + 48) = 42

180 - 42 = 138

138 ÷ 2 = 69

69 °

(Total for Question 11 is 3 marks)

12 (a) Simplify $(64p^9q^{12})^{\frac{2}{3}}$

$(\sqrt[3]{64})^2 = 4^2 = 16$

$p^{9 \times \frac{2}{3}} = p^6$

$q^{12 \times \frac{2}{3}} = q^8$

$16p^6q^8$

(2)

(b) Write as a single fraction $\frac{2}{3x} + \frac{4}{5x} - \frac{9}{10x}$

Give your answer in its simplest form.

$$\frac{2 \times 10 + 4 \times 6 - 9 \times 3}{30x}$$

$$= \frac{20 + 24 - 27}{30x}$$

$$= \frac{17}{30x}$$

$\frac{17}{30x}$

(2)

(c) Expand and simplify $4x(x - 5)(2x + 3)$
Show your working clearly.

$4x(2x^2 + 3x - 10x - 15)$

$= 4x(2x^2 - 7x - 15)$

$= 8x^3 - 28x^2 - 60x$

$8x^3 - 28x^2 - 60x$

(3)

(Total for Question 12 is 7 marks)

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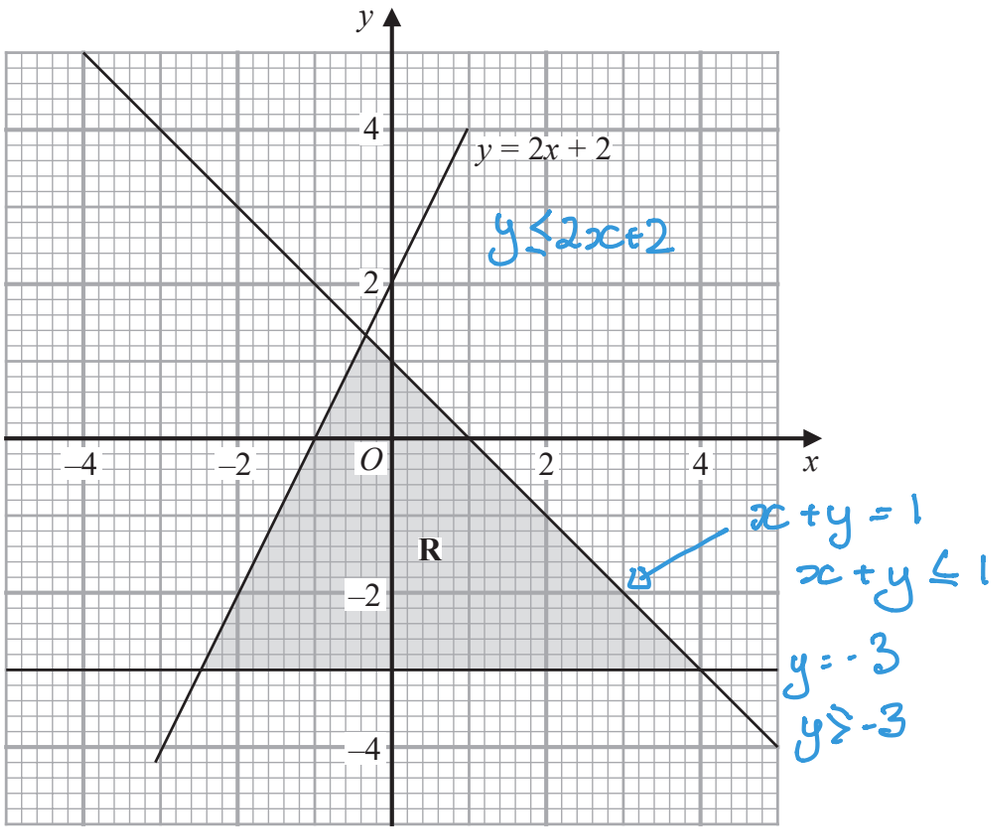
13



The region **R**, shown shaded in the diagram, is bounded by three straight lines.

Write down the three inequalities that define **R**.

$y \geqslant -3$

$x + y \leqslant 1$

$y \leqslant 2x + 2$

(Total for Question 13 is 3 marks)

14 Manuel collected information about the flights that arrived late at an airport last month.

The table gives information about the number of minutes that these flights were late.

Minutes late (L minutes)	Frequency
$0 < L \leqslant 10$	8
$10 < L \leqslant 15$	13
$15 < L \leqslant 25$	19
$25 < L \leqslant 40$	24
$40 < L \leqslant 60$	6

$\frac{6}{40}$

Frequency density

$8 \div 10 = 0.8$

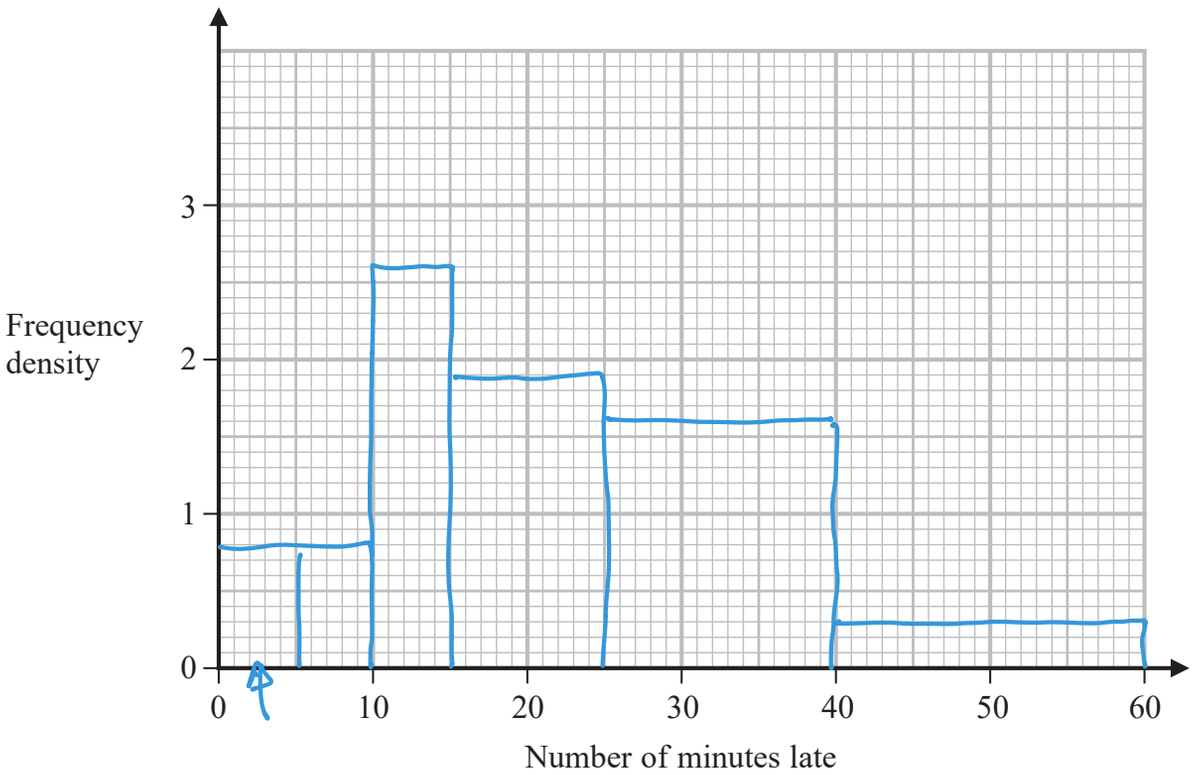
$13 \div 5 = 2.6$

$19 \div 10 = 1.9$

$24 \div 15 = 1.6$

$6 \div 20 = 0.3$

(a) On the grid, draw a histogram for this information.



(3)

Manuel selected at random a flight that was late by 25 minutes or less from his results.

(b) Work out an estimate for the probability that this flight was late by 5 minutes or less.

$5 \times 0.8 = 4$

$\frac{4}{40}$

.....

(2)

(Total for Question 14 is 5 marks)

15 The functions f and g are such that

$$f(x) = 2x - 3$$

$$g(x) = \frac{x}{3x + 1}$$

(a) State the value of x that cannot be included in any domain of g

$3x+1=0 \quad x=-\frac{1}{3}$

$-\frac{1}{3}$

(1)

(b) Find gf(x)
Simplify your answer.

$gf(x) = \frac{2x-3}{3(2x-3)+1} = \frac{2x-3}{6x-9+1}$

gf(x) = $\frac{2x-3}{6x-8}$

(2)

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

$y = \frac{x}{3x+1}$

$y(3x+1) = x \rightarrow 3yx + y = x$

$3yx - x = -y \rightarrow x(3y-1) = -y$

$x = -\frac{y}{3y-1} \qquad g^{-1}(x) = \frac{-x}{3x-1}$ or $\frac{x}{1-3x}$

$g^{-1}(x)$ = ...

(3)

(Total for Question 15 is 6 marks)

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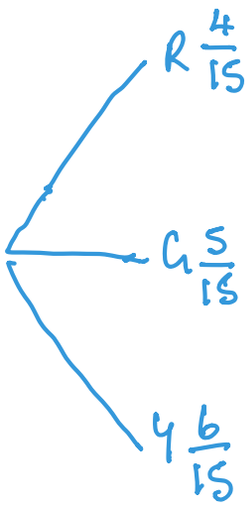
16 A box contains 15 counters.

There are 4 red counters, 5 green counters and the rest are yellow counters.

Niklas takes at random a counter from the box and writes down the colour of his counter.
He then puts the counter back into the box.

Sasha then takes at random a counter from the box and writes down the colour of her counter.

Work out the probability that the counters taken by Niklas and Sasha both have the same colour.



$$P(R,R) = \frac{4}{15} \times \frac{4}{15} +$$

$$P(G,G) = \frac{5}{15} \times \frac{5}{15} +$$

$$P(Y,Y) = \frac{6}{15} \times \frac{6}{15}$$

$$= \frac{16}{225} + \frac{25}{225} + \frac{36}{225}$$

$\frac{77}{225}$

(Total for Question 16 is 3 marks)

17 Express $\frac{8}{\sqrt{5}-1}$ in the form $\sqrt{a}+b$ where a and b are integers.

Show each stage of your working clearly.

$$\frac{8}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{8\sqrt{5}+8}{\sqrt{5}\sqrt{5}+\sqrt{5}-\sqrt{5}-1}$$

$$= \frac{8\sqrt{5}+8}{5-1} = \frac{8\sqrt{5}+8}{4}$$

$$= 2\sqrt{5}+2 \quad \text{not } \sqrt{a}+b$$

so $\sqrt{4}\sqrt{5}+2$

$= \sqrt{20}+2$

$\sqrt{20}+2$

(Total for Question 17 is 3 marks)

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18 Here is a quadrilateral *ABCD*.

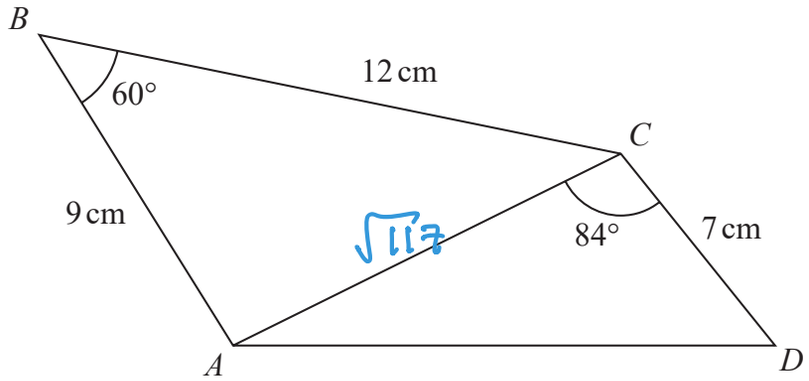

Diagram **NOT** accurately drawn

Calculate the area of quadrilateral *ABCD*.
Give your answer correct to 3 significant figures.
Show your working clearly.

$AC^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \times \cos 60$

$= 117$

$AC = \sqrt{117}$

Area $\frac{1}{2} \times 9 \times 12 \sin 60 + \frac{1}{2} \sqrt{117} \times 7 \times \sin 84$

$= 27\sqrt{3} + 37.650...$

$= 84.416...$

3sf

84.4 ..... cm^2

(Total for Question 18 is 5 marks)

19 The straight line **L** has equation $x - y = 3$
The curve **C** has equation $3x^2 - y^2 + xy = 9$

L and **C** intersect at the points P and Q.

Find the coordinates of the midpoint of PQ.
Show clear algebraic working.

L $x = y + 3$

$x^2 = (y+3)(y+3)$

$= y^2 + 6y + 9$

C $3(y^2 + 6y + 9) - y^2 + y(y+3) = 9$

$3y^2 + 18y + 27 - y^2 + y^2 + 3y - 9 = 0$

$3y^2 + 21y + 18 = 0$

$\div 3$ $y^2 + 7y + 6 = 0$

$(y+1)(y+6) = 0$

-1 -6

$x = -1 + 3$ $x = -6 + 3$

$= 2$ $= -3$ $(2, -1)$ $(-3, -6)$

Midpoint $(-0.5, -3.5)$

(-0.5 , -3.5)

(Total for Question 19 is 6 marks)

20 Here are the first four terms of an arithmetic series.

$$k \quad \frac{3k}{4} \quad \frac{k}{2} \quad \frac{k}{4}$$

$d = -0.25k$

Given that the 15th term of the series is $(90 + 2k)$,

calculate the sum of the first 30 terms of the series.

$S_n = \frac{n}{2}[2a + (n-1)d]$

15th term

$-0.25k \times 15 + 1.25k = 90 + 2k$

$-2.5k = 90 + 2k$

$4.5k = -90$

$k = \frac{-90}{4.5} = -20$

so $d = -0.25 \times -20$

$= 5$

S_{30}

$S_{30} = \frac{30}{2}(2 \times -20 + (30-1)5)$

$= 15(-40 + 145)$

$= 15 \times 105$

$= 1575$

1575

(Total for Question 20 is 5 marks)

21 The curve **C** has equation $y = f(x)$ where $f(x) = 9 - 3(x + 2)^2$
The point A is the maximum point on **C**.

(a) Write down the coordinates of A.

(-2 , 9)

(1)

The curve **C** is transformed to the curve **S** by a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(b) Find an equation for the curve **S**.

$y = 9 - 3(x - 4 + 2)^2$

$y = 9 - 3(x - 2)^2$

(1)

The curve **C** is transformed to the curve **T**.
The curve **T** has equation $y = 3(x + 2)^2 - 9$

(c) Describe fully the transformation that maps curve **C** onto curve **T**.

Reflection in the x-axis

(1)

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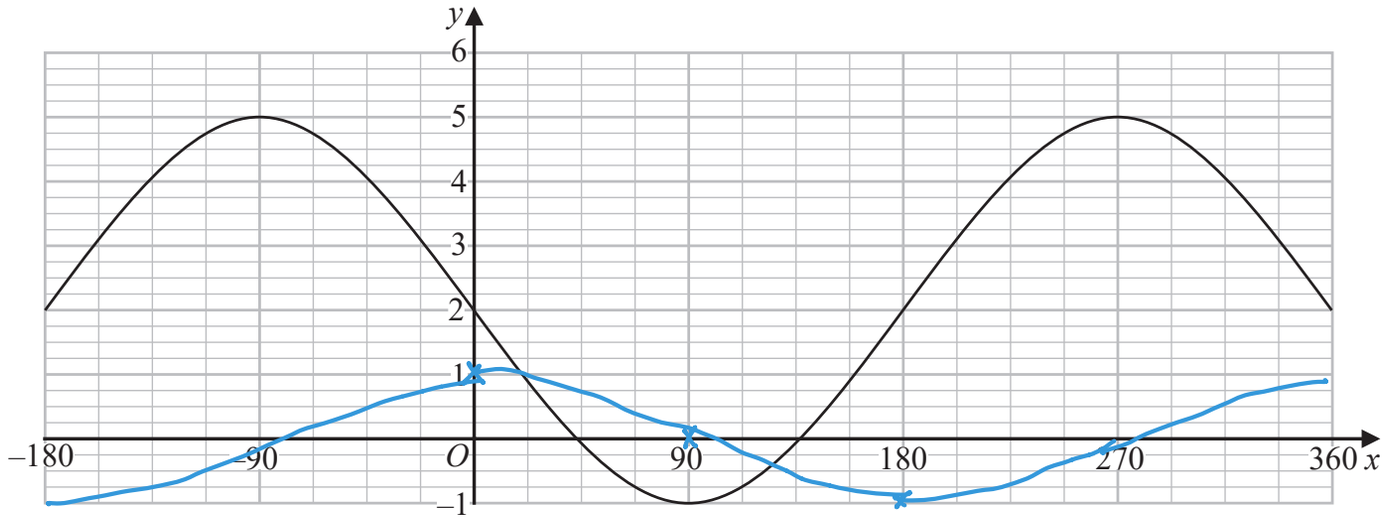
The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ 3

$b =$ -90

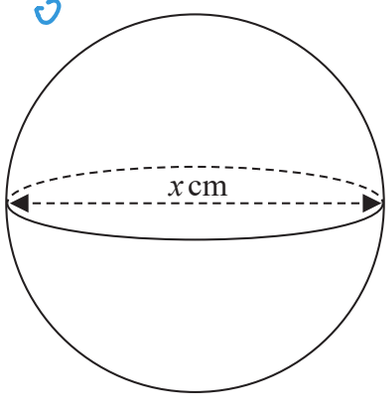
$c =$ 2

(3)

(Total for Question 21 is 6 marks)

22 The diagram shows a sphere of diameter x cm and a pyramid $ABCDE$ with a horizontal rectangular base $BCDE$.

$\frac{4}{3}\pi r^3$



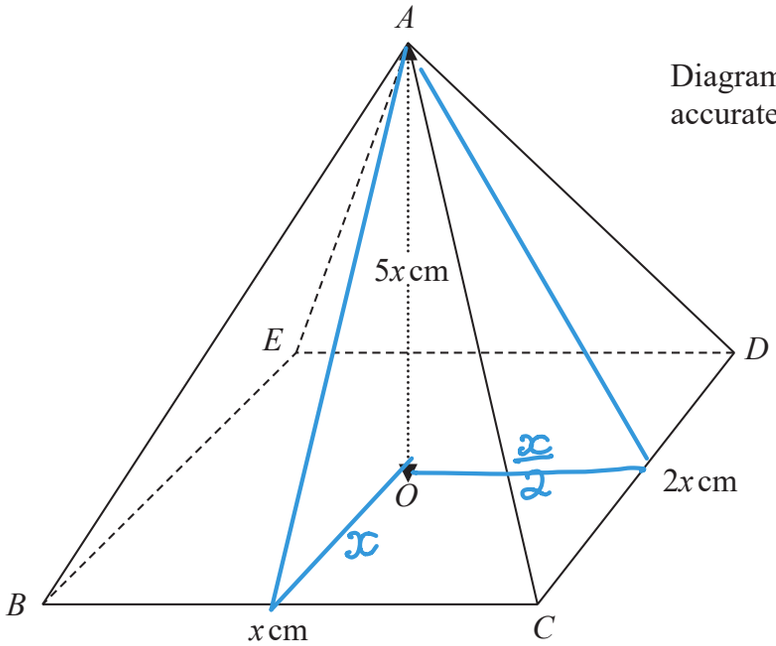


Diagram **NOT** accurately drawn

The vertex A of the pyramid is vertically above the centre O of the base so that $AB = AC = AD = AE$.

$BC = x$ cm, $CD = 2x$ cm and $AO = 5x$ cm.

The volume of the sphere is 288π cm^3

Calculate the total surface area of the pyramid.
Give your answer correct to the nearest cm^2

Sphere

$288\pi = \frac{4}{3}\pi r^3$

$\frac{288 \times 3}{4} = r^3$

$r = \sqrt[3]{216}$

$r = 6$ so $x = 12$

$x = 12$

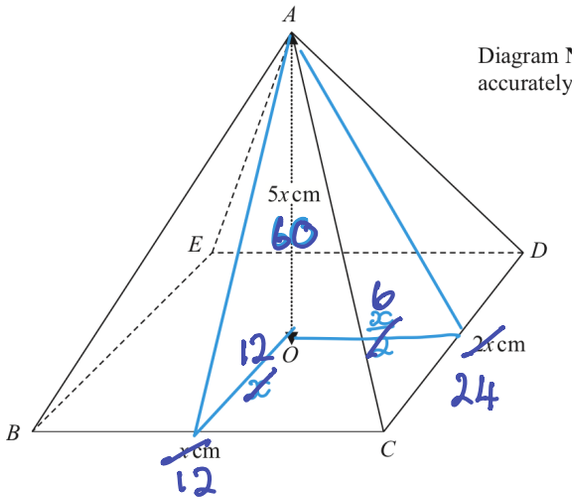
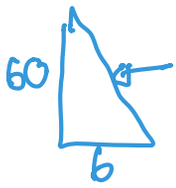


Diagram N
accurately

Surface area.



$\sqrt{60^2 + 6^2}$

$= 6\sqrt{101}$

$\sqrt{60^2 + 12^2}$

$= 12\sqrt{26}$

Area.

$12 \times 24 + 2 \times \left(\frac{1}{2} \times 24 \times 6\sqrt{101}\right) + 2 \times \left(\frac{1}{2} \times 12 \times 12\sqrt{26}\right)$

$= 2469.440...$

2469 cm^2

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS